McDiarmid's Inequality

- suppose:
  - $f(z_1, \ldots, z_m)$ real-valued
  - changing z_i changes f by at most c_i
  - i.e., $\forall z_1, \ldots, z_m, z_i'$:

$$\left| f(z_1, \ldots, z_i, \ldots, z_m) - f(z_1, \ldots, z_i', \ldots, z_m) \right| \leq c_i$$

  - $Z_1, \ldots, Z_m$ independent, not necessarily identical
- then

$$\Pr[\, f(Z_1, \ldots, Z_m) \geq \mathrm{E}[f(Z_1, \ldots, Z_m)] + \epsilon \,] \leq \exp\left(\frac{-2\epsilon^2}{\sum_{i=1}^m c_i^2} \right)$$

- e.g.: $f(Z_1, \ldots, Z_m) = \frac{1}{m} \sum_{i=1}^m Z_i$ [with $Z_i \in [0, 1]$]
  - then $c_i = 1/m$
  - get Hoeffding